

Calculus with *MATHEMATICA*[®]

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■ Preface

MATHEMATICA[®] is a powerful and elegant tool for exploring essentially every branch of mathematics. But in particular, its ease of use and its outstanding graphics capabilities make it an ideal laboratory for learning calculus. Too often, however, when *Mathematica* is used in a calculus course, it is presented merely as a collection of seemingly magical “commands” that draw pretty pictures or take care of tedious algebraic tasks. We believe that a more in-depth study of *Mathematica* itself can significantly enhance the experience of learning calculus, as well as being a highly worthwhile endeavor for its own sake.

It is a common refrain among teachers of calculus that the *concepts* of calculus are not difficult for students to grasp and that the primary causes of failure among calculus students are inadequate algebraic skills and inadequate understanding of precalculus concepts relating to functions. Thus computer software systems such as *Mathematica* become attractive as potential facilitators of success. After all, *why must students be able to perform tedious algebraic tasks when “the machine” can do it for them?* We believe that is the wrong question to ask, especially when asked rhetorically. A better question is: *Can the machine be used in a way that helps students gain skills that are as valuable and important as the algebraic skills that they may lack?* We think the answer is *yes* and that those skills can be attained through *learning how the machine works*. *Mathematica*, on one level, is just a machine that does exactly what skilled mathematicians do by hand on paper—only *much* faster. So understanding how *Mathematica* works—and how to use it well—may be viewed as a skill that is at least on par with being prolific at algebraic manipulation with pencil and paper. But the emphasis must be on understanding how *Mathematica* works, not just on knowing the names and syntax of a few “commands.” This is where *Mathematica* is superior to its competitors. It is designed around a single unifying idea (that everything is represented as an *expression*), and its inner workings are highly transparent and extraordinarily well documented.

So What Is This?

This interactive book is a unique approach to the modernization of the calculus course. Rather than “reforming” calculus by watering down its algebraic and computational content, we attempt to modernize it through the use of *Mathematica*. For instance, we do not completely exclude topics such as integration techniques and graphing with derivatives, which are sometimes mistakenly viewed as irrelevant when software can carry out such tasks. We emphasize integration by parts and the “reverse chain rule”, *i.e.*, simple substitution. (There are many elementary integrals that *Mathematica* cannot find without user intervention and the judicious use of the reverse chain rule.) We also emphasize conceptual understanding of the effect that derivatives have upon the shape of a graph and illustrate these ideas with animations.

This book is complete enough to be used as a text for a highly *Mathematica*-focused, three-semester calculus sequence. It can also be used as a supplement to a more traditional text in courses that have a *Mathematica* lab component. It should also be useful as an introduction to *Mathematica* with a review of calculus.

An extensive *Mathematica* tutorial is included, in which we strive to bring the student to a high level of competency—or an intermediate level of expertise. Hyperlinks provide easy and convenient access to information about *Mathematica* functions and to relevant parts of *The Mathematica Book*. In addition, extensive use is made of a collection of custom animations, graphics, and functions provided by the **MmaCalc** package. These are especially useful in the chapters on multivariable calculus.

The MmaCalc Package

Accompanying this book is a *Mathematica* package named **MmaCalc** that provides more than fifty functions designed to illuminate and facilitate important topics. Among them are sixteen special-purpose graphics functions and seventeen functions that create animations. All of them are described in detail—along with numerous examples of their use—in the documentation that is available through your Help Browser after the package is installed. Also among the documentation available in the Help Browser after installation of the **MmaCalc** package is a *Beginner’s Glossary of Mathematica Terminology*.

How To Use This Book

As you've already seen, the book is designed to be used through the *Mathematica* Help Browser. Hyperlinks display reference material in non-saveable pop-up notebooks rather than in the Help Browser, which allows you to view reference material without losing your place in the book. Hyperlinks must be activated with the button [Click here to activate hyperlinks](#), which appears at the top of all pages.

Input cells in the book are shaded in yellow and are intended to be entered (sequentially) by the reader. (When a package is needed, it is usually loaded once within each section—often in the first example of the section. So if you skip over an example, subsequent examples might not work properly.)

The header of each subsection of exercises contains a button like this: [Exercises](#), which when clicked will open a new notebook containing those exercises.

Remarks on Problem Solving

Teachers of elementary mathematics often place much emphasis upon George Polya's famous principles of problem solving:

- 1) Understand the problem.
- 2) Think of a plan.
- 3) Carry out the plan.
- 4) Look back.

We should think about these principles in the context of learning calculus with *Mathematica*. What role, if any, can *Mathematica* play in each of these four steps?

1. *Mathematica* can help us understand problems (and basic concepts) through graphics and animations. In many cases, it can be used “experimentally” to obtain a rough preliminary solution, which can then be refined into a more rigorous one.
2. *Mathematica* cannot think of a plan for us, but it does allow us to experiment easily with different approaches.
3. This is the most obvious place where *Mathematica* plays an important role by making difficult computations routine.
4. The graphics and computational capabilities provided by *Mathematica* can be very useful in looking back to see whether a solution is correct. However, using *Mathematica* does not diminish the importance of the “human role.” It cannot be emphasized enough that an answer isn't necessarily correct just because *Mathematica* has given it to you. Train your nose to do a “smell test” on every answer; in other words, *think* about whether the answer is reasonable. Moreover, even when your plan does produce a correct solution, by looking back you will often think of a better plan that produces a more “elegant” solution.

We offer the following variation on Polya's framework, modified for the context of working with *Mathematica*.

- *Understand* the problem.
- *Investigate* and *experiment*, both numerically and graphically.
- Devise a *more precise approach* based upon sound principles.
- Perform the appropriate *calculations*.
- Apply the *smell test*. *Check* algebraically and/or numerically. *Illustrate* graphically.
- *Reconsider* the method.

Computers are useless; they can only give you answers.

Pablo Picasso